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Students' beliefs and attitudes toward mathematics across time: A longitudinal examination of the theory of planned behavior☆

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ABSTRACT

The theory of planned behavior (TPB) offers a theoretically meaningful framework for examining students' beliefs and attitudes toward mathematics at school. However, longitudinal investigations of mathematics beliefs and attitudes using the TPB are scarce at best. To redress this imbalance, we examined the predictive validity of mathematics beliefs and attitudes, modeled using the four key constructs of the TPB (i.e., intention, attitude, norms, and control), on mathematics grades across time, while simultaneously controlling for quantitative reasoning. Furthermore, we explored the longitudinal interplay among these key constructs of the TPB. The total sample, drawn from various US middle schools, comprised 752 students at Time 1 and 514 students at Time 2. We used structural equation modeling to address the proposed research questions, and found that intention was associated with students' grades over time above and beyond quantitative reasoning. Additionally, intention at Time 1 was positively associated with control at Time 2, whereas – after controlling for shared variances – attitude at Time 1 showed a negative relation with control at Time 2. Intention and norms were reciprocally related across time. The current study provides the first longitudinal support for the validity of a mathematics beliefs and attitudes model strongly rooted in the TPB.

1. Introduction

At the individual, local, and national levels, mathematics proficiency has been acknowledged as key for personal and economic success (see e.g., Geary, 1996). In light of this, former President Obama made it a priority during his presidency to foster science and mathematics achievement in K-12 education in order to increase the number of students who pursue careers in the highly paid and highly rewarded fields of science, technology, engineering, and mathematics (STEM; whitehouse.gov/issues/education/k-12/educate-innovate [accessed 08/25/2016]). Therefore, the question of which factors can help to predict students' achievement in mathematics and, moreover, their likelihood of further engagement with STEM fields is of importance for educational policy and practice.

The current study focused on students' beliefs and attitudes toward mathematics as a potential facilitator of students' engagement in mathematics. For this purpose, we conceptualized mathematics beliefs and attitudes in terms of the Theory of Planned Behavior (TPB; Ajzen, 1991). We also conducted what we believe to be the first longitudinal test of this model, with two broad aims. First, we investigated the extent that mathematics beliefs and attitudes, as conceptualized by the TPB, could explain mathematics grades as indicators of student achievement in this domain. To this end, we examined the predictive validity of the mathematics beliefs and attitudes components that are part of the TPB framework on changes in students' mathematics grades across time, while simultaneously controlling for students' underlying quantitative reasoning skills. Second, we examined the longitudinal interplay between these different components of mathematics beliefs and attitudes across time, specifically, intention, attitude, subjective norms, and perceived behavioral control (in the remainder of this article, we

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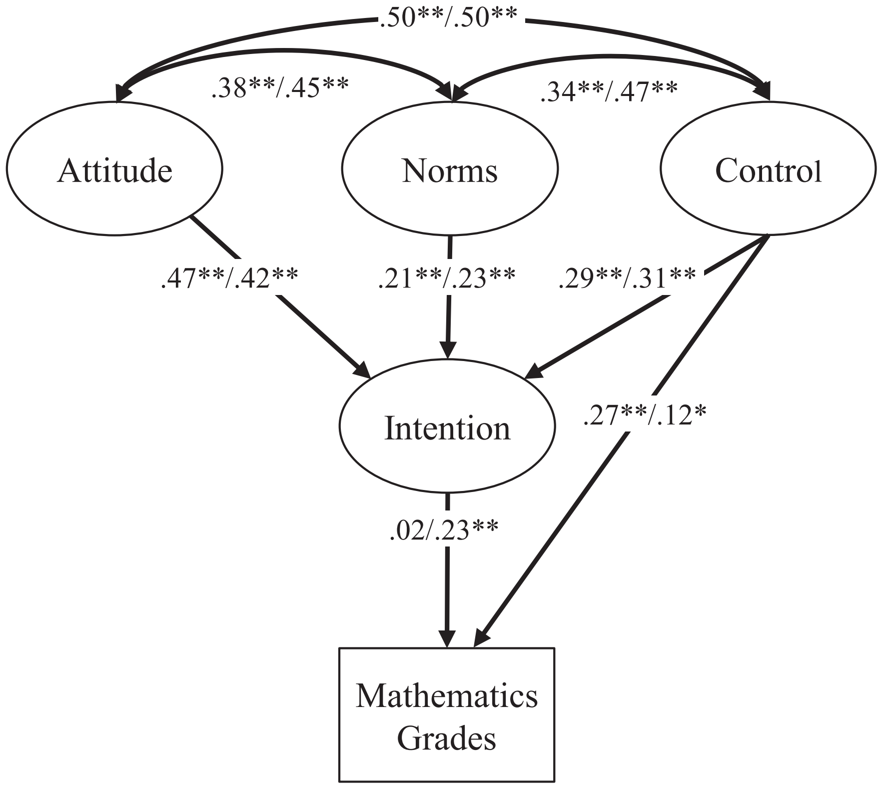

Fig. 1. Model A1 and Model A2: Mathematics belief and attitude components conceptualized in terms of the theory of planned behavior (i.e., Intention, Attitude, Norms, and Control) predicting mathematics grades. For the sake of clarity, control variables and residual variances are not shown in the path diagram. Results for June 2012 (Model A1 for T1) are depicted before the slashes, for November 2012 (Model A2 for T2) after the slashes.
† $p < 0.10$. * $p < 0.05$. ** $p < 0.01$.

subsume all four components under the term *TPB-based mathematics beliefs and attitudes*). In addition, we investigated potential reciprocal links between TPB-based mathematics beliefs and attitudes and grades with the goal of obtaining insight into their developmental dynamics.

1.1. The theory of planned behavior

The TPB aims to explain and predict behavior by focusing on four components (Ajzen, 1991, 2012; for a visualization of the TPB, see Figs. 1 and 2): an individual's (a) intention to carry out a specific behavior as well as the person's (b) attitude (i.e., personal evaluation of a behavior), (c) subjective norms (i.e., perceived social pressures to perform a behavior), and (d) perceived behavioral control (i.e., competence perceptions) with respect to the behavior in question. In particular, the TPB[1] argues that a person's intention to carry out a certain behavior is the best predictor of his or her actual performance of that behavior. This intention, in turn, is determined by the three other components. First, individuals' beliefs about the outcomes of a behavior and their evaluations of these outcomes compose their *attitude* toward that behavior. Second, normative beliefs about the expectations of others and the motivation to comply with these comprise the *subjective norms* against which a behavior is compared. Third, control beliefs about facilitative and inhibitive factors and the perceived power of these factors determine the amount of *perceived behavioral control* over performing a behavior. The TPB assumes that planned behavior is not imperatively motivated behavior; rather, intentions to perform a behavior can be predominantly caused by an attitude *or* subjective norms, whereas a sufficient degree of actual behavioral control, in turn, is necessary to carry out any intention (Ajzen, 1991, 2012). Furthermore, according to the TPB, behavior is directly determined not only by intention but also by control (with control also influencing behavior indirectly via intention; Ajzen, 1991).

[1] The following description is adapted from Ajzen (1991, 2012).

1.2. Mathematics beliefs and attitudes and their relationship with performance

Numerous studies have shown that beliefs and attitudes toward mathematics predict mathematics achievement (Ma & Kishor, 1997). Several of these studies have predicted mathematics achievement by drawing from theoretical models that are similar to the TPB. For example, Eccles and colleagues' expectancy-value model (Eccles et al., 1983) is an extensive model that predicts academic performance and choice. Among the central components of the model are: (a) "subjective task values", which are measured in part by items that reflect attitudes, such as a student's perception of how much he or she likes an academic subject and also its perceived usefulness/importance, and (b) "ability self-perceptions", which are measured by items that reflect a student's belief in his or her ability to do well in a subject and his or her expectations about success in that subject. Thus, ability self-perceptions are somewhat analogous to perceived behavioral control in the TPB model. Each of these two components of the model has been shown to predict mathematics achievement. For instance, in a two-year longitudinal study on seventh to ninth grade students, Meece, Wigfield, and Eccles (1990) found that ability self-perceptions in Year 1 predicted Year 2 mathematics grades. In another two-year study of 200 eighth through tenth graders, subjective task values predicted mathematics grades in Year 1 (for males only) and ability self-perceptions predicted mathematics grades in both Years 1 and 2 (for both males and females; Eccles, Adler, & Meece, 1984).

A related line of research is the work on academic self-concept (see, e.g., for an overview: Marsh, 2006). Academic self-concept is typically described as the mental representation of one's academic ability (e.g., Brunner et al., 2010; Niepel, Brunner, & Preckel, 2014) and is therefore "conceptualized as students' beliefs of their own domain-specific and/or global academic capabilities" (Pinxten, Marsh, De Fraine, Van Den Noortgate, & Van Damme, 2013 p. 2). Research on the multidimensionality of widely employed measures of mathematics self-concept found that a two-dimensional model of self-concept fit better than a one-dimensional model (Pinxten et al., 2013). The two dimensions

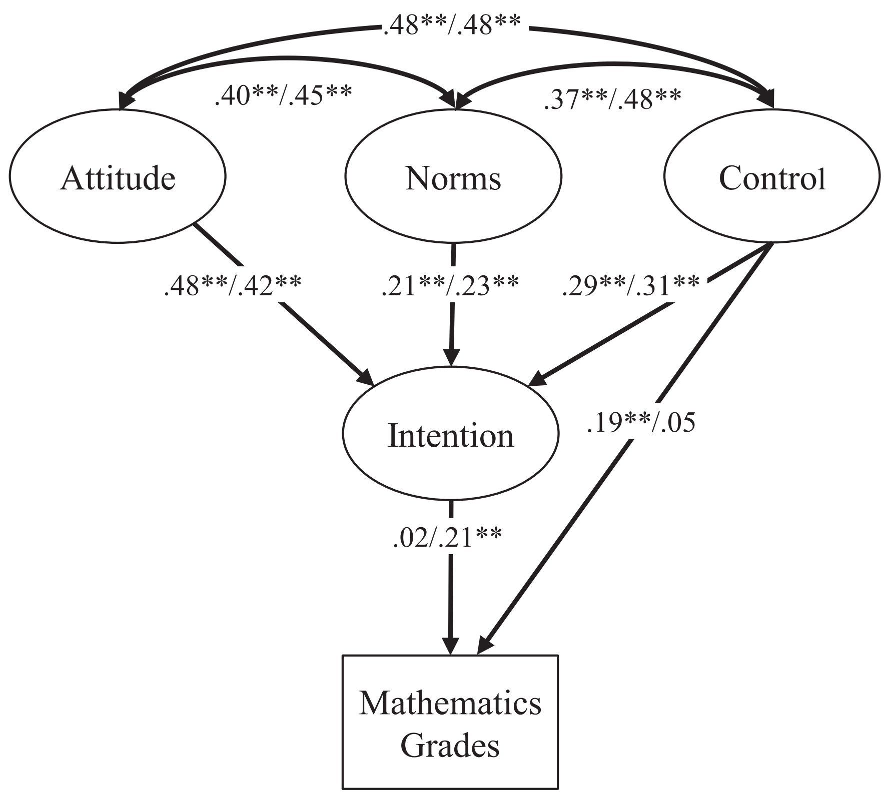

Fig. 2. Model B1 and Model B2: Mathematics belief and attitude components conceptualized in terms of the theory of planned behavior (i.e., Intention, Attitude, Norms, and Control) predicting mathematics grades while controlling for students' reasoning test scores. Notably, students' reasoning test scores were included into the analysis as predictor of all depicted variables but, for simplicity, are not shown. For the sake of clarity, control variables and residual variances are not shown in the path diagram. Results for June 2012 (Model B1 for T1) are depicted before the slashes, for November 2012 (Model B2 for T2) after the slashes.
† $p < 0.10$. * $p < 0.05$. ** $p < 0.01$.

corresponded to mathematics competence (analogous to perceived behavioral control in the TPB model), and enjoyment of mathematics (analogous to mathematic attitudes in the TPB model; Pinxten et al., 2013; see also Arens, Yeung, Craven, & Hasselhorn, 2011). Furthermore, a longitudinal study of over 4000 third to seventh graders in Belgium found that both mathematics enjoyment and mathematics competence beliefs were positively related to mathematics achievement over time (Pinxten et al., 2013). However, the effect of competence beliefs was much larger, and the effect of enjoyment disappeared when competence beliefs were controlled for. Similarly, a study of nearly 2000 third to sixth grade students in Germany found support for a multidimensional model of mathematics self-concept, with dimensions corresponding to “mathematics affect” and “mathematics competence” (Arens et al., 2011). As in the Pinxten et al. (2013) study, mathematics competence was a stronger predictor of mathematics achievement than mathematics affect, although mathematics affect was a significant predictor as well.

To provide another example of a theory that has similarities to the TPB, the self-system model of motivational development (Skinner, Kindermann, Connell, & Wellborn, 2009) has been used to predict educational outcomes with several components, including “self” and “engagement” components. The “self” components include academic self-concept, whereas the “engagement” components include, for example, positive attitudes toward school. Clearly, both of these are related to the attitude and control components of the TPB. Given the previous discussion of attitudes and self-concept, one would predict that “self” and “engagement” components should also be predictive of academic achievement. This prediction was verified in a one-year longitudinal study of over 1800 seventh through ninth graders in Australia (Green et al., 2012). Specifically, various sources of motivation (i.e., beliefs about school and school work) at Time 1 predicted positive attitudes toward school at Time 2. These positive attitudes then predicted behavioral engagement with school, as indexed by class participation, homework completion, and absenteeism. Finally, behavioral engagement predicted test performance in spelling and mathematics.

In sum, various theories posit that attitudes and self-beliefs should predict student achievement in mathematics, and a great deal of research has shown that this is indeed the case. Moreover, these constructs predict achievement over time. This provides evidence that at least two of the components of the TPB, attitudes and perceived behavioral control, should be predictive of mathematics achievement over time. Although previous work has powerfully predicted mathematics achievement with these constructs, one goal of research is to continue to improve our ability to predict important outcomes, and thus to test various theories' ability to do just that. One theory that holds promise in improving our predictive capacity is the TPB. The TPB, which has been shown to predict behavior across many domains (see for one meta-analysis: Armitage & Conner, 2001), includes two additional components not extensively studied by the theories reviewed above, subjective norms and intention. Intention is posited to be the single best predictor of behavior in the model (Ajzen, 1991), and subjective norms are one of the three predictors of intention. In an effort to improve our ability to predict mathematics achievement, it is important to fully test the predictive power of all of the components of the TPB over time, rather than simply a subset of them. The extant research that has investigated the relationship between the TPB and mathematics achievement is reviewed below.

1.3. Examining mathematics beliefs and attitudes using the theory of planned behavior

Figs. 1 and 2 visually represent the TPB mathematics beliefs and attitudes model as originally introduced by Lipnevich, MacCann, Krumm, Burrus, and Roberts (2011): Intention in mathematics precedes mathematics achievement as result of behaviors (i.e., mathematics grades as markers for achievement) and is itself determined by mathematics-related attitude,[2] subjective norms, and perceived

[2] Please note that the term *attitude* is meant here as one out of four TPB-based mathematics beliefs and attitudes, together with *norms*, *control*, and *intention*.

behavioral control (see Lipnevich et al., 2011). In two cross-sectional studies, Lipnevich et al. (2011) found that 62.6% (in a U.S. sample) and 65% (in a Belarusian sample) of the variance in intention were explained by attitude, norms, and control. Furthermore, the latent relation between intention and mathematics achievement (teacher-assigned grades as a behavioral outcome) was found to be significant and large in magnitude for both samples ($\beta = 0.64$ in the U.S. sample and $\beta = 0.90$ in the Belarusian sample). Unexpectedly, the path from control to achievement was not significant in the U.S. sample ($\beta = 0.01$) and was even significantly negative in the Belarusian sample ($\beta = -0.28$; Lipnevich et al., 2011). However, with regard to the model-based latent correlations, all TPB-based mathematics beliefs and attitudes were found to be significantly positively related to achievement and explained a considerable amount of its variance in both samples (25% in the U.S. sample and 28% in the Belarusian sample). Recently, in drawing on cross-sectional data, Burrus and Moore (2016) and Lipnevich, Preckel, and Krumm (2016) provided further evidence for the predictive link between students' TPB-based mathematics and their mathematics achievement. Burrus and Moore (2016) found TPB-based mathematics beliefs and attitudes to predict students' achievement test scores in mathematics independently of students' background (e.g., socioeconomic status), GPA in mathematics courses, and conscientiousness. Similarly, Lipnevich et al. (2016) found TPB-based mathematics beliefs and attitudes to incrementally predict students' grades beyond reasoning and personality (i.e., Big Five). In addition, in a study of 220 12- to 15-year-old students, Hagger and colleagues found that attitude and subjective norms predicted intention, which then predicted both mathematics grades and mathematics homework behavior, while perceived behavioral control directly predicted both outcomes (Hagger, Sultan, Hardcastle, & Chatzisarantis, 2015). However, as outlined in the next section, further research on TPB-based mathematics beliefs and attitudes is clearly necessary in order to more fully understand the mechanisms underlying the relation between TPB-based mathematics beliefs and attitudes on the one hand and achievement on the other hand.

1.4. *Examining TPB-based beliefs and attitudes toward mathematics across time*

Although longitudinal research on mathematics beliefs and attitudes testing different theoretical frameworks (see, for example, the studies described above) has been conducted, longitudinal research on the TPB-based mathematics beliefs and attitudes model (with its unique components; e.g., subjective norms and intention) has yet to be conducted. Such studies would not only indicate whether the TPB model predicts achievement over time, they would also likely provide information as to whether the components of the model interact over time. In fact, longitudinal studies are needed to complement recent cross-sectional research on TPB-based mathematics beliefs and attitudes, as only longitudinal studies give researchers a window into developmental dynamics across time. Extending our knowledge of the developmental dynamics of the TPB-based mathematics beliefs and attitudes components, in turn, would have manifold practical as well as theoretical implications. To date, we know virtually nothing about the longitudinal relations among TPB-based mathematics beliefs and attitudes and their common interplay with mathematics grades across time. For example, it is an open question whether the attitude, norms, and control components are longitudinally independent (as suggested by Fishbein & Ajzen, 2010, with regard to the general TPB framework) or meaningfully associated—concurrently, these components have been shown to be moderately interrelated (Lipnevich et al., 2011). Furthermore, it is not known whether mathematics grades (as a behavioral outcome) and subsequent TPB-based mathematics beliefs and attitudes are reciprocally related across time. Longitudinal research on TPB-based mathematics beliefs and attitudes extends previous research in this domain by determining whether mathematics beliefs and attitudes, as conceptualized by TPB, predict *changes* in students' mathematics grades (i.e., after taking their preceding mathematics grades into account), or in other words, whether positive TPB-based mathematics beliefs and attitudes are potentially linked to an improvement in students' mathematics grades across time. Last but not least, examining TPB-based mathematics beliefs and attitudes across time is also of importance for scholars interested in the TPB framework as a whole because research investigating the TPB framework across multiple measurement occasions is relatively scarce (for one example, see Courtois et al., 2014, on students' acceptance of tablet devices).

2. The present study

Building on the research of Lipnevich et al. (2011), we used the TPB to provide a framework for mathematics beliefs and attitudes. The current study substantially broadens the scope of analysis by adding a developmental perspective to existing research. To this end, we used two measurement points to examine the longitudinal interplay between TPB-based mathematics beliefs and attitudes (i.e., intention, attitude, norms, and control) and their contributions to explaining mathematics grades across time. We focused on two broad research aims:

(1) We aimed to demonstrate that TPB-based mathematics beliefs and attitudes explain students' mathematics grades across time over and above their underlying reasoning skills. We controlled for quantitative reasoning as a proxy for cognitive ability as one core predictor of grades, overall performance, and development in mathematics (Floyd, Evans, & McGrew, 2003; Roth et al., 2015; see also Strenze, 2007)—to examine the "net" associations between the TPB-based mathematics beliefs and attitudes and grades *independently* of individual differences in reasoning skills. By using two measurement points and measuring students' mathematics grades at the second measurement point (T2), we were also able to control for their initial mathematics grades at the first measurement point (T1; i.e., baseline level), enabling us to examine whether *changes* in mathematics grades (i.e., variance in mathematics grades that cannot be explained by students' baseline mathematics grades) can be explained by TPB-based mathematics beliefs and attitudes. Prior cross-sectional findings by Lipnevich et al. (2011, 2016) led us to expect that the various components of mathematics beliefs and attitudes encapsulated by the TPB would predict students' mathematics grades over and above their baseline mathematics grades and their underlying quantitative reasoning skills.

(2) We aimed to explore the longitudinal interplay between the four components of TPB-based mathematics beliefs and attitudes, as well as their potential reciprocal relations with mathematics grades, across time. Here, potential long-lasting interdependencies between attitude, norms, control, and intention might be revealed. In addition, we examined potential reciprocal relations between students' mathematics grades and TPB-based mathematics beliefs and attitudes across time.

3. Method

3.1. *Sample and procedure*

The data used in the present study were taken from two waves (henceforth referred as Time 1 [T1] and Time 2 [T2]) of a larger ongoing longitudinal project focusing on the development of noncognitive skills in students attending private, independent middle schools (i.e., Grade 6 to Grade 8) in the United States. Before this study was conducted, human subjects approval was attained from the Institutional Review Board at Educational Testing Service. The study was conducted in accordance with the ethical principles and guidelines outlined by the American Psychological Association. T1 took place in June 2012, T2 in November 2012. For a full description of the project, see the technical report by Petway, Rikoon, Brenneman, Burrus, and Roberts (2016). From this larger overall data set, we selected those students who provided valid data on TPB-based mathematics beliefs and attitudes at T1 and on reasoning. Furthermore, for T2, we selected only those students who had already participated at T1. Our resulting sample comprised 752 U.S. students (50.0% male) at T1 and 514 students at T2 (49.3% male), each of which was enrolled at one of eight independent middle schools in six U.S. states (i.e., Connecticut, Georgia, Kentucky, Massachusetts, New Jersey, and New York) with an average within-school sample size of 94 (range: 38 to 123 students). The self-reported average age of the sample was 12.57 years ($SD = 0.88$, Range: 11 to 15; $n = 751$) at T1 and 12.78 years ($SD = 0.68$, Range: 12 to 14; $n = 513$) at T2. At T1, 45.1% of the students reported being in Grade 6 (1.4% at T2), 34.6% in Grade 7 (52% at T2), and 20.3% in Grade 8 (46% at T2). Approximately 99% of the students at T2 ($n = 509$) reported their ethnicity (an ethnicity variable was not obtained at T1) as follows: Caucasian/European American (71.3%), Asian American (7.5%), African American (3.7%), Multiracial (3.3%), Latino/Hispanic (2.6%), Middle Eastern (2.2%), International (2.0%), Native American (0.2%); 7.3% of the students responded with “not sure”. Students responded to the survey during a scheduled class period that was arranged by a school coordinator. An entire class was tested at the same time, yet each individual student was given the option to decline to participate in the assessment. The entire assessment took approximately 40 min to complete (the actual TPB assessment took < 5 min of this time). Each student was assigned a unique ID number that was used to match students' data.

3.2. Variables and measures

3.2.1. TPB-based mathematics beliefs and attitudes (T1 and T2)

To assess TPB-based mathematics beliefs and attitudes at both measurement points, we used the Mathematics Attitude Questionnaire (MAQ; Lipnevich et al., 2011). The MAQ consists of a total of 22 items measuring attitude (five items, e.g., “I enjoy studying math”; “I think math is fun”), norms (five items; e.g., “Most of my friends think math is necessary to succeed”; “My friends think that math is an important subject”), control (five items; e.g., “If I invest enough effort, I can succeed in math”; “It is impossible for me to succeed in math”), and intention (seven items; e.g., “I am determined to become good at math”; “I will try to work hard to make sure I learn math”). Students responded to these items on a four-point Likert-type rating scale ranging from 1 (*never or rarely*) to 4 (*usually or always*).

3.2.2. Quantitative reasoning

Students' quantitative reasoning was measured by the quantitative reasoning scores from the Comprehensive Testing Program 4 (CTP 4), an assessment developed by the Educational Records Bureau (ERB, 2012) to collect basic information about students' performance in various academic areas. Quantitative reasoning is a subtest that focuses on logical, algebraic, geometrical, probabilistic, and statistical reasoning, classification, and recognition. For this project, the participating schools provided CTP scores for each individual student, represented as a percentile. This percentile was based on Independent School Norms offered by ERB. The quantitative reasoning scores used in the present study originated from the CTP 4 testing session conducted in Spring 2012, thus, close to T1.[3] According to ERB (2012), the internal consistencies across the three middle school grades (in independent schools) are $\alpha = 0.82$ for Grade 6, $\alpha = 0.88$ for Grade 7, and $\alpha = 0.88$ for Grade 8.

3.2.3. Mathematics grades (T1 and T2)

Mathematics grades were provided by participating schools at both measurement points. Thereby, the amount of missing data was higher for grades than all other measures: Whereas we had data from 752 students overall at T1 and 514 students at T2 who filled out the MAQ and the quantitative reasoning test (i.e., overall sample as described earlier), for mathematics grades, we had data from 512 students at T1 and 401 students at T2. Because schools provided grades in different formats (e.g., numeric scores, letter grades), all grades were converted into a scale ranging from 0 (corresponding to a failing mathematics grade) to 4.30 (corresponding to an A+ grade) with the scale increasing from 0 to 4.30 in increments of 0.3 or 0.4 (e.g., 3.0; 3.3; 3.7) as is commonplace for converting letters into numbers (see, e.g., collegeboard.com/html/academicTracker-howtoconvert.html [accessed 01/30/2017]).

3.3. Data analysis

We used SPSS 22 to calculate descriptive statistics for the measures. To address our research questions, we applied a structural equation modeling (SEM) approach with Mplus 7 (Muthén & Muthén, 1998–2012) using maximum likelihood (ML) estimation. We used (a) the Root Mean Square Error of Approximation (RMSEA), (b) the Comparative Fit Index (CFI), and (c) the Standardized Root Mean Square Residual (SRMR; Hu & Bentler, 1999; Kline, 2011) to evaluate model fit, with RMSEA ≤ 0.08, SRMR ≤ 0.10, and CFI ≥ 0.90 indicating acceptable fit and RMSEA ≤ 0.05, SRMR ≤ 0.08, and CFI ≥ 0.95 indicating good fit (Browne & Cudeck, 1993; Hu & Bentler, 1999). Chi-square was used only descriptively due to its sensitivity to sample size. To handle missing data, we applied multiple imputation based on an unrestricted variance-covariance baseline model including all measures of TPB-based mathematics beliefs and attitudes, mathematics grades, and quantitative reasoning as outlined in Asparouhov and Muthén (2010) to generate 5 complete data sets for the 752 students.

In our data, students were nested within eight schools. Intraclass correlations (ICC) for the applied measures ranged from 0.01 to 0.21 (i.e., with ICCs of ≤ 0.03 for all TPB-based mathematics beliefs and attitudes measures at both measurement points, of 0.03 and 0.2 for mathematics grades at T1 and T2 respectively, and of 0.21 for quantitative reasoning) pointing to the fact that ignoring the nested data structure might result in biased parameters and standard errors. We therefore controlled for the nested data structure by applying a fixed-effects design (see, e.g., Huang, 2016). Consequently, we calculated seven dummy-coded variables representing the eight different schools, with the most frequently attended school (i.e., $n = 123$ students) as the baseline group. These seven dummy-coded variables were included as control variables in all of our SEM analyses to account for all variability associated with the school level (Huang, 2016).

4. Results

4.1. Descriptive statistics

To provide initial insights into the data, we calculated bivariate manifest correlations, score means, and standard deviations for all measures at both measurement points using SPSS. Table 1 contains the results (above the diagonal). All TPB-based mathematics attitude components were found to be positively intercorrelated. Furthermore, all TPB-based mathematics components were positively related to mathematics grades and reasoning, with the exception of norms (T1 and T2) with reasoning (close to T1) and norms at T1 with mathematics grades (grades at T1).

[3] As we had reasoning test score data for only $N = 487$ students in Fall 2012 (close to T2) and as the two test scores were highly correlated ($r = 0.92$, $p < 0.001$), we used students' test scores from Spring 2012 only (close to T1) in our analyses to control for potential effects of students' quantitative reasoning skills.

Table 1
Observed correlations (above the diagonal), model-based correlations (below the diagonal), means (M), standard deviations (SD), and reliabilities in terms of McDonald's ω (in parentheses) for TPB-based mathematics beliefs and attitudes (attitude, norms, control, and intention; at T1 and T2), mathematics grades (T1 and T2), and reasoning test scores (close to T1).

Constructs	Correlations											*M*	*SD*
	1	2	3	4	5	6	7	8	9	10	11		
1. Attitude T1	(0.88)	0.68**	0.31**	0.22**	0.42**	0.23**	0.60**	0.47**	0.26**	0.24**	0.16**	2.33	0.75
2. Attitude T2	0.68**	(0.86)	0.24**	0.36**	0.36**	0.41**	0.46**	0.60**	0.19**	0.30**	0.20**	2.41	0.75
3. Norms T1	0.39**	0.29**	(0.81)	0.54**	0.32**	0.25**	0.42**	0.37**	0.05	0.10*	−0.02	2.87	0.62
4. Norms T2	0.27**	0.44**	0.57**	(0.83)	0.25**	0.42**	0.37**	0.49**	0.12*	0.21**	0.05	2.90	0.63
5. Control T1	0.50**	0.39**	0.34**	0.26**	(0.87)	0.48**	0.49**	0.36**	0.31**	0.31**	0.18**	3.27	0.68
6. Control T2	0.27**	0.50**	0.23**	0.46**	0.53**	(0.89)	0.34**	0.53**	0.21**	0.27**	0.22**	3.27	0.70
7. Intention T1	0.69**	0.51**	0.49**	0.40**	0.59**	0.42**	(0.90)	0.63**	0.23**	0.26**	0.08*	3.00	0.61
8. Intention T2	0.48**	0.67**	0.38**	0.56**	0.42**	0.63**	0.66**	(0.91)	0.23**	0.31**	0.12**	3.04	0.64
9. Grades T1	0.26**	0.19**	−0.02	0.07	0.28**	0.18**	0.17**	0.20**	(—)	0.68**	0.35**	3.37	0.61
10. Grades T2	0.23**	0.28**	0.05	0.17**	0.29**	0.26**	0.23**	0.30**	0.69**	(—)	0.41**	3.43	0.63
11. Reasoning	0.18**	0.18**	−0.06	0.03	0.20**	0.18**	0.11**	0.13**	0.42**	0.47**	(—)	57.08	30.57

Note. Observed correlation, means, and standard deviations were calculated with SPSS (using pairwise deletion). Results for model-based correlations derived from a baseline model which was specified to obtain model-based correlations (i.e., Model D).
* $p < 0.05$.
** $p < 0.01$.

4.2. Preliminary SEM analyses

4.2.1. Testing measurement models and factorial structure

Starting with SEM in order to isolate potential local misfit (Tomarken & Waller, 2003) and to examine whether intention, attitude, norms, and control were empirically distinguishable, we specified two confirmatory factor analyses (CFAs) – one for each measurement point – that included all four TPB-based mathematics beliefs and attitudes measures. Thereby, intention was modeled as a latent construct with seven indicators; attitude, norms, and control were modeled as latent constructs with five indicators each. Results indicated that the five negatively worded indicators (i.e., the MAQ consists of five negatively and 17 positively worded items) exhibited weak factor loadings, ranging from 0.01 to 0.29 at T1 and 0.07 to 0.22 at T2, with two of them failing to reach statistical significance (i.e., one factor loading per point of measurement). Model fit indicated a poor approximation to the data (i.e., CFI < 0.90; see Table 2). Therefore, we specified each CFA again with only the 17 positively worded indicators: that is, intention with five indicators, and attitude, norms, and control with four indicators each. Results related to this set of data showed that all four TPB-based mathematics beliefs and attitudes were adequately measured by their respective indicators and that the corresponding four-factorial structure fit the data well at both measurement points (see Table 2). Reliabilities in terms of McDonald's ω varied from 0.81 to 0.91 (see Table 1 on the diagonal). In subsequent analyses, the latent factors of TPB-based mathematics beliefs and attitudes were therefore specified with only the 17 positively worded indicators.

4.2.2. Establishing measurement invariance across time

In this phase of the analysis, we tested whether the latent measures of the TPB would capture the same target constructs across time (i.e., we tested for measurement invariance across time). Testing for measurement invariance is a prerequisite for tackling our research questions because at least metric measurement invariance must be established in order to interpret longitudinal autoregressive and cross-lagged coefficients. For this purpose, we applied a stepwise approach to test for measurement invariance (see e.g., Little, Preacher, Selig, & Card, 2007). In a first step, we conducted a longitudinal confirmatory factor analysis including all latent constructs, which served as a baseline model. In a second step, we tested this baseline model against a metric invariant model in which the size of the factor loadings was set to be invariant across time. Metric invariance was established if the metric invariant model exhibited similar fit indices as the baseline model (see Christ & Schlüter, 2012). To compare both models (i.e., testing the metric invariant against the baseline model), we conducted chi-square difference tests. In all longitudinal models, we allowed the residuals of the corresponding indicators to correlate across the two waves of measurement (i.e., correlated uniqueness; Little et al., 2007). An examination of the model fit indices (see Table 2) and nonsignificant difference-testing statistics between these variously restricted models ($\Delta\ \chi^2 = 11.19$,

Table 2
Model fit indices.

Model	χ^2	*df*	CFI	RMSEA (90% CI)	SRMR
Confirmatory factor analyses					
CFA at T1 (22 items)	1391.733	329	0.875	0.066 (0.062–0.069)	0.066
CFA at T2 (22 items)	1155.221	329	0.866	0.058 (0.054–0.061)	0.066
CFA at T1 (17 items)	648.752	204	0.943	0.054 (0.049–0.059)	0.048
CFA at T2 (17 items)	600.679	204	0.930	0.051 (0.046–0.056)	0.048
Measurement invariance across time					
Baseline model	1385.697	664	0.944	0.038 (0.035–0.041)	0.053
Metric invariant model	1396.891	677	0.944	0.038 (0.035–0.040)	0.054
TPB-based mathematics beliefs and attitudes per measurement point					
Model A1: TPB-implied structure at T1	693.325	219	0.938	0.054 (0.049–0.058)	0.049
Model A2: TPB-implied structure at T2	610.255	219	0.931	0.049 (0.044–0.053)	0.047
Model B1: TPB-implied structure at T1 (controlled for reasoning)	723.442	232	0.939	0.053 (0.049–0.057)	0.047
Model B2: TPB-implied structure at T2 (controlled for reasoning)	623.793	232	0.934	0.047 (0.043–0.052)	0.046
Main analyses					
Model C: two-wave cross-lagged model	1489.935	755	0.945	0.036 (0.033–0.039)	0.052
Model D: baseline model (to obtain model-based correlations)	1489.699	755	0.945	0.036 (0.033–0.039)	0.052

Note. *df* = degrees of freedom; CFI = comparative fit index; RMSEA = root mean square error of approximation; SRMR = standardized root mean square residual.

$\Delta df = 13$, $p = 0.595$) suggested that a model with metric invariance provided a reasonable approximation of the data. Thus, subsequent longitudinal analyses were conducted on models that imposed metric invariance.

4.2.3. Testing TPB-based mathematics beliefs and attitudes at each measurement point

Here we modeled the predictive structure of mathematics beliefs and attitudes as implied by the TPB and their assumed relation to mathematics grades separately for each measurement point. We did so to thoroughly investigate whether the latent structure as implied by the TPB held at each wave and to test whether intention and control were cross-sectionally related to mathematics grades before integrating all measures into one longitudinal cross-lagged model (as described below). Therefore, we specified a model in which mathematics grades (modeled as a manifest construct) were predicted by intention (modeled as a latent construct) and by control (modeled as a latent construct) for each wave of measurement. In turn, intention was predicted by attitude, norms, and control (all modeled as latent constructs). Fig. 1 depicts the results for both models, which we called Model A1 and A2. Fit indices suggested acceptable model fit (see Table 2). In both models, the predictive paths from the latent independent variables (i.e., attitude, norms, and control) to intention, as well as the latent correlations between the latent independent variables, were statistically significant ($p < 0.001$). Attitude, norms, and control jointly explained 58% and 60% of the variation in intention at T1 at T2 respectively. At T1, the path from control to mathematics grades was significant ($p < 0.001$), and at T2, the paths from intention ($p < 0.001$) and control ($p = 0.037$) to mathematics grades were significant. Together, intention and control explained 8% and 10% of the variation in students' mathematics grades at T1 and T2 respectively.

In addition, we investigated whether the established TPB-implied structural model would hold after controlling for students' quantitative reasoning skills at each measurement point. For this purpose, we included the quantitative reasoning test score (modeled as a manifest construct) in our analyses as a predictor of all latent mathematics beliefs and attitudes constructs as well as mathematics grades (i.e., Models B1 [for T1] and B2 [for T2]). Fig. 2 shows the results. Model B1 and B2 demonstrated acceptable fit (see Table 2). Reasoning was significantly (with $p < 0.001$) associated with attitude ($\beta = 0.21$ and $\beta = 0.20$ for T1 and T2, respectively), control ($\beta = 0.23$ and $\beta = 0.21$ for T1 and T2), and mathematics grades ($\beta = 0.43$ and $\beta = 0.50$ for T1 and T2) but not with intention or norms at either measurement point ($p > 0.05$). Overall, the TPB-implied structure remained relatively stable after controlling for reasoning; however, the path between control and mathematics grades at T2 differed in that the link failed to reach statistical significance ($p = 0.355$). Reasoning, attitude, norms, and control jointly explained 62% and 63% of the variation in intention at T1 and T2 respectively. Reasoning, intention and control jointly explained 24% of the variation in students' mathematics grades at both T1 and T2. In the subsequent longitudinal analyses, we used a model that controlled for quantitative reasoning (modeled as a manifest construct).

4.3. Main analyses: examining students' TPB-based mathematics beliefs and attitudes across time

Finally, we addressed our two research aims by means of a two-wave cross-lagged SEM (Little et al., 2007), which we called Model C. In Model C, the temporal precedence of the examined variables was purely implied by the longitudinal design, which enabled us to analyze the longitudinal interplay between all four TPB-based mathematics beliefs and attitudes components and mathematics grades while controlling for individual differences in quantitative reasoning. Fig. 3 illustrates Model C; Table 2 shows the overall model fit. In Model C, reasoning was included as a predictor of all latent variables as well as of mathematics grades. At both measurement points, reasoning was associated with control ($\beta = 0.23$, $p < 0.001$ and $\beta = 0.10$, $p = 0.031$ for T1 and T2) and mathematics grades ($\beta = 0.47$, $p < 0.001$ and $\beta = 0.25$, $p < 0.001$ for T1 and T2) and showed no association with norms ($p > 0.05$). Reasoning was significantly related to both intention and attitude only at T1 ($\beta = 0.12$, $p = 0.004$ and $\beta = 0.21$, $p < 0.001$) but not at T2 ($p > 0.05$).[4]

To obtain model-based correlations, we specified a baseline model of Model C (i.e., Model D) in which all latent measures of TPB-based mathematics beliefs and attitudes and manifest measures of grades and reasoning were correlated, while controlling for the nested data structure (as was done in all SEM analyses). The resulting coefficients ρ and corresponding *p*-values are depicted in Table 1 (below the diagonal); the overall model fit of Model D is shown in Table 2.

4.3.1. First research question: predicting students' mathematics grades across time

TPB-based mathematics beliefs and attitudes were significantly related to subsequent achievement. As depicted in Fig. 3, intention at T1 showed a positive and significant direct link to mathematics grades at T2 (i.e., changes in students' mathematics grades) above and beyond students' grades at T1 and students' quantitative reasoning test scores, whereas control at T1, norms at T1, and attitude at T1 did not. Furthermore, model-based correlations (see Table 1; below the diagonal) indicated that intention, attitude, and control at T1 were significantly positively related to subsequent mathematics grades (T2) with a magnitude of 0.23 or greater (with $p < 0.001$), whereas norms at T1 were not associated with subsequent mathematics grades at T2 ($p > 0.05$).

4.3.2. Second research question: exploring the longitudinal interplay between TPB-based mathematics beliefs and attitudes

All autoregressive paths across time were positive and significant (with $p < 0.001$; see Fig. 3). The paths from intention to subsequent control, as well as to subsequent norms, showed a statistically significant positive association (with $p < 0.001$), indicating that intention was related to a higher perception of competence in mathematics (control) as well as perceiving important others as feeling more favorably toward mathematics (norms). Additionally, we found a significant positive association between norms at T1 and intention at T2 (with $p = 0.030$), indicating that norms were associated with a stronger intention to succeed in mathematics (intention). Furthermore, we did not find strong empirical evidence for relations between students' mathematics grades and their subsequent TPB-based mathematics beliefs and attitudes at a later point in time. Indeed, only a marginally significant link between grades at T1 and intention at T2 (with $p = 0.056$; see Fig. 3) emerged. Finally, the path from attitude to subsequent control showed a statistically significant negative association (with $p = 0.036$), pointing to a suppression effect due to shared variances with concurrent measures (as the latent correlations between attitude at T1 and control at T2 were shown to be significant and positive with $\rho = 0.27$ [$p < 0.001$]; see Table 1).

5. Discussion

The present study was conducted to address the lack of previous longitudinal research on mathematics beliefs and attitudes using the

[4] In response to a comment from an anonymous reviewer, we performed our main analyses a second time in which all variables (i.e., control, attitude, intention, grades, and reasoning) were modeled as manifest constructs. In such, we tested whether intention at T1 still predicted grades at T2 beyond quantitative reasoning, when both constructs (and not only intention) were specified as manifest constructs (i.e., not controlled for respective measurement errors). We found virtually no changes in the resulting longitudinal relation between intention and subsequent grades (as depicted in Fig. 3): Intention at T1 still predict grades at T2 beyond quantitative reasoning with a resulting coefficient of $\beta = 0.10$ ($p = 0.023$; versus $\beta = 0.12$, $p = 0.032$, as in the original Model C as presented in Fig. 3).

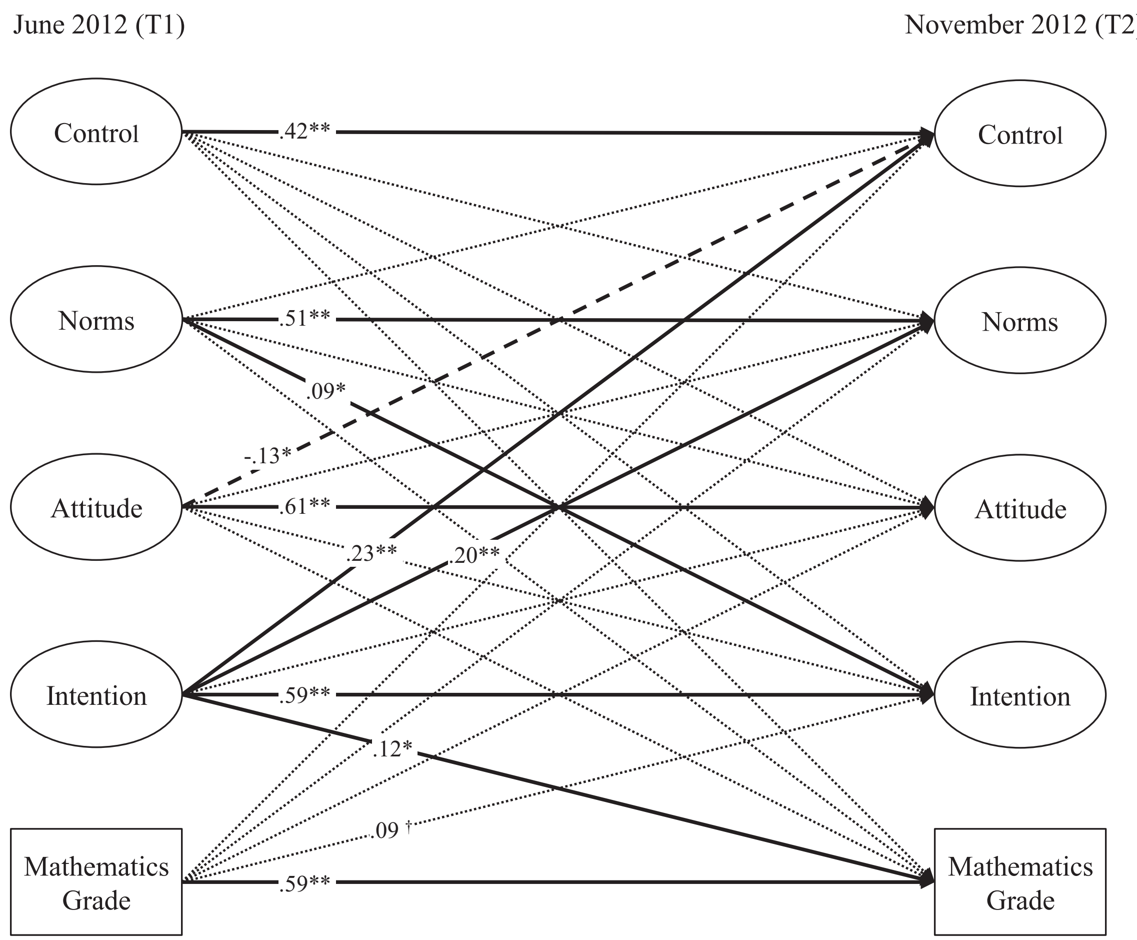

Fig. 3. Model C: Two-wave cross-lagged model between mathematics belief and attitude components conceptualized in terms of the theory of planned behavior (i.e., Intention, Attitude, Norms, and Control) and mathematics grades while controlling for students' reasoning test scores. Notably, students' reasoning test scores were included into the analysis as predictor of all depicted variables but, for simplicity, are not shown. For the sake of clarity, control variables, residual variances, and correlational paths were omitted from the path diagram and only standardized parameter estimates with $p < 0.10$ are shown. Positive associations are depicted by continuous lines, negative by dashed lines, and nonsignificant with $p > 0.05$ by dotted lines.
† $p < 0.10$. * $p < 0.05$. ** $p < 0.01$.

theory of planned behavior (TPB; Ajzen, 1991, 2012; see also Lipnevich et al., 2011). In sum, we found evidence that the TPB components of intention and control were concurrently related to mathematics grades and that intention was longitudinally related to changes in mathematics grades beyond students' reasoning skills. Moreover, in exploring the longitudinal interplay between the four components of TPB-based mathematics beliefs and attitudes, we found that intention was positively related to subsequent control and that intention and norms were reciprocally related. Furthermore, attitude showed a negative relation with subsequent control when shared variances with concurrent measures were controlled for. In the following sections, we discuss our findings in greater detail.

5.1. Predicting students' mathematics grades across time

Students' intention to succeed in mathematics at T1 was shown to be positively associated with subsequent mathematics grades at T2—beyond students' grades at T1, concurrent TPB-based mathematics beliefs and attitudes (i.e., attitudes, norms, and control at T1), and quantitative reasoning skills. In our cross-sectional analyses, we additionally found that intention was predicted by concurrent attitudes, norms, and control. Overall, the results of the present study were in line with the TPB in showing that intention was directly linked to changes in mathematics grades and that concurrent attitudes, norms, and control substantially predicted intention. However, control at T1 provided no incremental validity in directly predicting subsequent grades at T2. Furthermore, in line with previous research (Roth et al., 2015), students' quantitative reasoning test score was substantially related to mathematics grades at both time points.

The results are consistent with cross-sectional research predicting mathematics achievement with the TPB model (Burrus & Moore, 2016; Lipnevich et al., 2011; Lipnevich et al., 2016). They are also consistent with previous research using other theoretical frameworks that have predicted mathematics achievement by means of attitudes (measures of intrinsic value or affect) and control (measures of competence perceptions) (e.g., Arens et al., 2011; Green et al., 2012; Meece et al., 1990; Pinxten et al., 2013) in that attitudes and control as measured at T1 were positively related to grades as measured at T2 on a bivariate level (model-based correlations). Notably, our results diverge somewhat from previous research in that the effects of these variables were analyzed after controlling for intention, which was shown to be the best predictor of grades at T2 (besides reasoning). Further, we found that norms predicted intention not only concurrently but also longitudinally. Thus, even after controlling for students' intention at T1, their peers' attitudes and perceived control toward mathematics (i.e., subjective norms) still seem to affect their intention in mathematics several months later. Collectively, these findings speak to the importance of including both subjective norms and intention in any attitudinal model and indicate that modifications may be needed to other attitudinal theories often regarded as more established in the educational literature.

5.2. Exploring the longitudinal interplay between TPB-based mathematics beliefs and attitudes

The present study is the first to examine longitudinal autoregressive and cross-lagged interrelations between TPB-based mathematics beliefs and attitudes. As longitudinal research on the dynamics between the components of the TPB framework (i.e., attitude, subjective norms, perceived behavioral control, intention, and behavioral outcomes) is generally scarce, our results may be of interest not only to scholars and practitioners who deal with beliefs and attitudes toward mathematics but also to those interested in the TPB framework on a general level. In other words, several findings from the present study enable a better understanding of the developmental dynamics of not only TPB-based mathematics beliefs and attitudes but also regarding the TPB framework as a whole.

We found that intentions were positively associated with subsequent perceived behavioral control. Hence, stronger intentions to succeed in mathematics seem to strengthen a person's perceived behavioral control vis-à-vis doing well in mathematics. A plausible explanation of this finding is offered by Gollwitzer and Kinney's (1989) work on implemental mindsets and the tendency for individuals to overestimate their control (illusion of control; Langer, 1975). Hence, the mindset of an individual who wants to implement a chosen goal (an intention to perform a behavior) is assumed to lead to better accessibility of cognitions that generally favor goal striving (the implementation of one's intention; Taylor & Gollwitzer, 1995). In sum, having formed an intention to perform well seems to benefit the accessibility of control beliefs that foster one's perceived behavioral control. At the same time, students' perceptions of how much they like mathematics (i.e., attitude) seem to buffer this effect of intention on control somewhat: After controlling for shared variance of concurrent measures (mainly intention and reasoning), students' attitudes were negatively associated with subsequent perceived behavioral control. Thus, while stronger intentions to succeed in mathematics seem to foster unrealistically optimistic perceptions of control, students' perception of liking mathematics seems to simultaneously disfavor such illusionary control beliefs. One may therefore assume that students who enjoy studying mathematics tend to develop more realistic control beliefs than students who do not. Furthermore, intention was found to be associated with subsequent subjective norms, which could also be explained by the effects of implemental mindsets: Besides better accessibility of cognitions that favor goal striving (as mentioned earlier), individuals pursuing a certain goal tend to additionally blunt out distracting stimuli that could prevent them from reaching their goal (see also Brandtstädter, 2009). We realize that these interpretations are post hoc and that transferring findings from research on implemental mindsets to the TPB framework is speculative in nature and requires further investigation—however, we find the initial evidence to be promising.

5.3. Limitations and future directions

In Models A1 to B2, we used cross-sectional designs to test whether the latent structure as implied by the TPB held at each wave. As grades were provided by schools, the employed TPB-based survey (i.e., MAQ) was thus used to retrospectively predict mathematics grades in Models A1 to B2. However, in Models A1 to B2, we modeled the structure of TPB-based mathematics beliefs and attitudes as implied by the TPB and interpreted all of the observed relations in accordance with the TPB. This implies that the nature of the relation between intention and behavior is causal and that attitude, norms, and control constitute a person's intention, which, in turn, predicts behavior (i.e., mathematics grades as behavioral outcomes). Our longitudinal data base enabled us to calculate a cross-lagged SEM (Model C) in which all variables were temporally ordered by design, thereby collecting longitudinal evidence on the predictive validity of mathematics beliefs and attitudes operationalized in terms of the TPB. In interpreting our results, however, readers should keep in mind that cross-sectional and even longitudinal designs do not allow causal inferences to be made; intervention studies are therefore called for.

A further limitation is that control and intention showed an inconsistent pattern in statistically predicting grades cross-sectionally. Specifically, at T1 perceived behavioral control was the superior predictor of mathematics grades (Models A1 and B1), whereas at T2 intention was the superior predictor (Model A2 and B2). Future research should therefore thoroughly replicate and extend our study to evaluate TPB-based mathematics beliefs and attitudes and their relative contribution in predicting grades (as well as further criteria; see paragraph below) more thoroughly. To this end, future longitudinal research should include more than two waves of measurement, which would allow researchers to analyze whether intention mediates predictive paths from control, norms, and attitude on mathematics grades longitudinally (i.e., longitudinal mediation analysis; see, e.g., MacKinnon, 2008). By means of such an analysis researchers could not only investigate the longitudinal structural relations of TPB-based mathematics beliefs and attitudes as claimed by the theory but also examine its direct and indirect longitudinal effects on grades to further clarify the respective contribution of each TPB construct in predicting subsequent grades. Moreover, more than two waves of measurement would allow researchers to study patterns of growth in students' achievement, beliefs, and attitudes (e.g., through latent curve models; see, e.g., Bollen & Curran, 2006).

Our sample was quite selective, as it comprised only U.S. middle school students enrolled in private schools. Therefore, future research aimed at generalizing our findings across different school types (e.g., public schools), different age groups (e.g., elementary school children), and countries outside the U.S. are needed as well.

The current study built upon Lipnevich et al.'s (2011) work and extended it longitudinally. In following Lipnevich et al.'s approach, we related students' mathematics beliefs and attitudes (as captured by the MAQ) to mathematics grades as criteria (Lipnevich et al., 2011; see also Burrus & Moore, 2016; Lipnevich et al., 2016). We thus used the TPB to predict grades, although the TPB was designed to predict behavior. Although grades are certainly related to a number of behaviors in mathematics-specific contexts - such as doing homework, studying for exams, paying attention in mathematics classes, or seeking the support of parents, peers, and teachers - they are not pure measures of behavior (Burrus & Moore, 2016). Moreover, grades are only proxies for students' achievement, as they are also dependent on characteristics beyond the student, for instance, teacher characteristics. Given these limitations, our statistical models may arguably underestimate the hypothesized relation between mathematics beliefs and attitudes on the one hand and mathematics achievement on the other hand. Future research should therefore use the TPB to predict specific behaviors such as concrete learning-related efforts in mathematics to obtain a more fine-grained picture on the validity of TPB-based mathematics beliefs and attitudes. Similarly, students' choice of specialized classes in high school, majors in college, or careers in early adulthood would be additional behavioral criteria to further test the validity of the TPB in the domain of (mathematics) education. Clearly there is a need to further explore the nomological network of TPB-based mathematics beliefs and attitudes.

5.4. Conclusion

The present results provide further support for the application of the TPB in the educational domain—namely, in the domain of mathematics. One major advantage of embedding mathematics beliefs and attitudes in the TPB is its utility in planning and conducting interventions. Because attitudes, norms, and control all predict intention, and because intention predicts mathematics grades longitudinally, it may be inferred that interventions that influence each of these components may also have a downstream effect on grades (recall, however, the caveat stated above that intervention studies are needed). That is, each TPB component can serve as a starting point for practical interventions: changing students' attitude, control, or norms should change their intentions (Fishbein & Ajzen, 2010). For example, attitude interventions have been outlined in the social psychological literature, focusing on influencing basic human needs (Cialdini & Goldstein, 2004), and in the educational psychology literature through utility-value interventions that emphasize the importance of mathematics for students' lives (Hulleman, Godes, Hendricks, & Harackiewicz, 2010). Perceived behavioral control can be improved by interventions that are designed to increase self-efficacy. This includes teaching effective learning strategies, stressing effort rather than success or failure, creating facilitative attributions, and helping students form positive self-evaluations (Margolis & McCabe, 2004). Finally, to the extent that attitudes and perceived control are improved on a large-scale basis, subjective norms might also be improved. That is, as students see their peers' attitudes and perceived control toward mathematics improve, their perception of the norms surrounding the value of mathematics might also change. Overall, practitioners can rely on the extensive research in the fields of social and educational psychology to design future mathematics beliefs and attitudes interventions. By publishing our results, we hope to stimulate further research, and possibly educational policy, in this field.

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